

# Data Recovery in Differentially Encoded Quadrature Phase Shift Keying

John D. Bard  
 Mohamed K. Nezami  
 Mary Ellen Diaz  
 Mnemonics, Inc.  
 Melbourne, Florida

## Abstract

Information in a PSK modulation format is contained in the phase. For Gray-coded DEQPSK, this information is contained in the change in phase. With advances in DSP technology, fundamental trigonometric signal processing functions which have, in the past, been unavailable now provide a mechanism for recovering phase differential directly from the phase plane. This paper presents a new algorithm for recovering step phase changes for a DEQPSK modulated signal by computing the phase angle from the arctangent of the ratio  $Q/I$ , and comparing it with the previous phase. This phase difference is then mapped to a monotonic metric for the purpose of soft-decision Viterbi decoding.

## Introduction

Traditional integrate and dump demodulators have been forced, by circumstances, to map phase changes of DEQPSK signals into the I-Q plane. Hardware implementations had no access to fundamental trigonometric functions, while early DSP applications were denied their availability because of their expense in terms of system resources. However, the advent of sophisticated, high speed DSP technology has made these functions available and practical to use in real-time applications, eliminating the need for I-Q plane translation henceforth enabling recovery of the phase differential directly from the phase plane. Clever use of mapping techniques facilitates this recovery of this information into 3-bit soft decision format suitable for Viterbi decoding.

## Differentially Encoded Quaternary Phase Shift Keying

In PSK modulated waveforms the signal is not the amplitude value of I or Q, but the instantaneous phase represented by I and Q. In theory, a phase-magnitude pair map unambiguously to I and Q values. For a constant envelope information signal such as a PSK modulation, variations in the amplitude can be construed as noise. There is no radial component of the signal that is due to information. For Differentially Encoded QPSK (DEQPSK) modulation, the information is contained in the phase differential. The main reason to differentially encode a QPSK signal is to avoid ambiguity when performing

carrier and baud tracking. A fading component would affect both I and Q, but the effect on the phase phase rotation is minimized. For flat Rayleigh channels, where the variation due to fading is very slow compared to the duration of the symbols, the phase perturbation can be considered constant when differencing the phase of two consecutive signals.

The Gray Code-mapping for DEQPSK in MIL-STD-188-183 is presented in Figure 1 below:

Carrier Phase Advance	Data Pair	
	I	Q
0	0	0
$\pi/2$	0	1
$\pi$	1	1
$3\pi/2$	1	0

Figure 1 Gray Code Mapping

This maps to 3-bit soft decisions show in Figure 2.

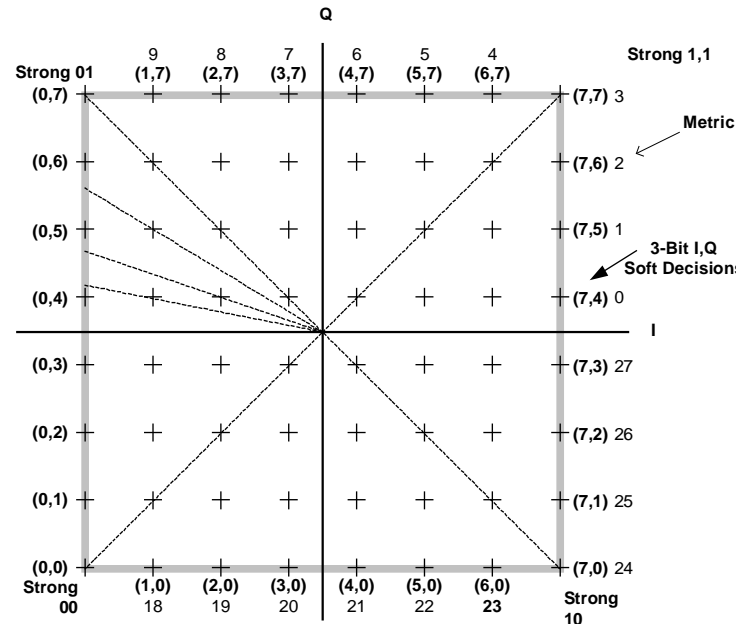


Figure 2 Three-bit Soft Decisions

The phase angle for a given IQ pair can be calculated directly using the arctangent function and is given by:

$$\phi = \tan^{-1}(Q/I) \quad (1)$$

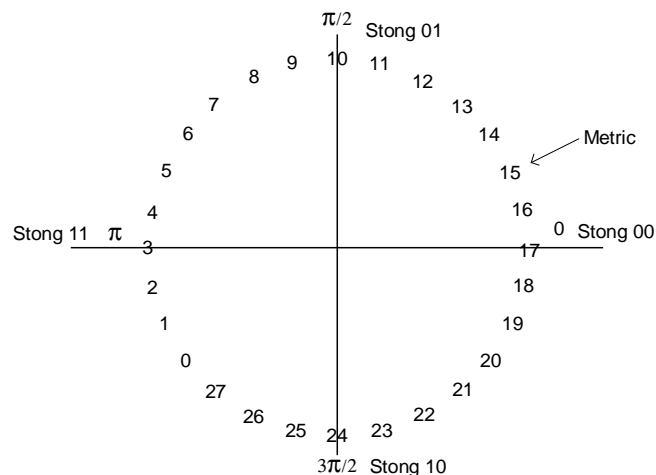
Trigonometric functions available in DSP math libraries such as the atan2() allow this extremely straightforward approach. Run-time robustness is guaranteed by built-in detection and handling of the divide by zero condition and real-time execution system integrity is preserved due to the fact that the algorithm runs in a fixed number of cycles. During information recovery, the phase angle is calculated once per baud. In a system running at a sample rate of 16,000 samples per baud, the minimum processor performance required to effect this calculation in the real time would be given by:

$$\frac{\text{Instructions}}{\text{Baud}} * \text{Symbol Rate} \quad (2)$$

For a DEQPSK signal with an over the air symbol rate of 16000 sps being demodulated by a system running on an Analog Devices ADSP 21000 family SHARC DSP with a guaranteed worst case arctangent execution of 82 instruction cycles [1]. This translates into a processor bandwidth of 1.32 MIPs well within the performance range of today's DSP technology.

### Mapping

Once the phase angle is obtained, the difference is determined through comparison with the phase angle computed for the previous baud. This difference is then reduced modulo  $2\pi$  to map onto the unit circle shown in Figure 3. The continuous phase difference is then mapped to a quantized metric, 0 through 28. The figure shows the relationship between quantized phase difference metric and soft-decisions as guided by Figure 1. As an example, a phase difference of 3 is a strong "11" or 77 as a 3-bit soft-decision pair. As you migrate clock-wise from that point, the I decision gets weaker. So a phase difference of 5 would map to a soft-decision pair of 57.



**Figure 3 Phase Mapping**

Referring to Figure 1, a simple look-up table maps this metric to 3-bit soft data for I and Q to be presented for Viterbi decoding. Note that calculation of the phase differential directly from the phase plane and reserving mapping to the I-Q plane until the completion of this calculation results in the generation of the least ambiguous values for the soft decisions.

The soft decision in one arm is guaranteed to be one end of the range or the other (either a zero or a 7). Traditional integrate and dump methods, where the phase differential is recovered from the I-Q plane can result in soft decision I-Q values contained anywhere within the area outlined by the shaded gray square in Figure 1. This technique effectively filters the effect of any variations in the signal amplitude due to noise by ensuring the soft decisions will lie directly within the shaded band in Figure 1.

### Error Signal

The soft decisions are then converted into data decisions which are used to generate the reference signal used in the phase lock loop to drive the error signal to zero. The standard decision map for QPSK is given by in Figure 4. For data decision aided error recovery, the error signal is derived as follows:

IQ	Phase (radians)
11	$\pi/4$
01	$3\pi/4$
00	$5\pi/4$
10	$7\pi/4$

**Figure 4 QPSK Decision Mapping**

With an unknown phase error  $\phi_e$ , the corresponding set of equations for I and Q is then

$$I = \cos(\phi_e + \pi/4 + k\pi/2) \quad (3)$$

$$Q = \sin(\phi_e + \pi/4 + k\pi/2) \quad , k=0,1,2,3 \quad (4)$$

Consider the case where  $k=0$ . Some trigonometric manipulation yields the following set of equations,

$$I = \frac{\sqrt{2}}{2} (\cos(\phi_e) - \sin(\phi_e)) \quad (5)$$

$$Q = \frac{\sqrt{2}}{2} (\cos(\phi_e) + \sin(\phi_e)) \quad (6)$$

Assuming  $\phi_e$  is small, the linear combination, Q-I isolates  $\phi_e$

$$Q - I = \sqrt{2} \phi_e \quad (7)$$

Figure 5 extends this for all data decisions for the QPSK error signal

K	IQ	error signal $\phi_e =$
0	11	$\frac{Q - I}{\sqrt{2}}$
1	01	$-\frac{Q + I}{\sqrt{2}}$
2	00	$-\frac{Q - I}{\sqrt{2}}$
3	10	$\frac{Q + I}{\sqrt{2}}$

**Figure 5 QPSK Phase Error by Data Decision**

Once the data decision is made the corresponding error signal equation is used to compute the error signal in the baud time.

### For Comparison, an Optimal Method

An optimal implementation [2] of soft bit generation from a DEQPSK modulated signal based on a maximum likelihood decoder yields the following:

$$P(I = 1) = \frac{P_A}{P_A + P_B} \quad P(Q = 1) = \frac{P_C}{P_C + P_D} \quad (8)$$

Where,  $P_A$  is the probability of vector pair in the IQ plane conditioned on the data value of I being 1

$$P_A = \cosh(2\eta(I_0 - Q_1))\cosh(2\eta(I_1 + Q_0)) + \cosh(2\eta(I_0 - I_1))\cosh(2\eta(Q_0 - Q_1)) \quad (9)$$

$P_B$  is the probability of vector pair in the IQ plane conditioned on the data value of I being 0

$$P_B = \cosh(2\eta(I_0 + Q_1))\cosh(2\eta(I_1 - Q_0)) + \cosh(2\eta(I_0 + I_1))\cosh(2\eta(Q_0 + Q_1)) \quad (10)$$

$P_C$  is probability of vector pair in the IQ plane conditioned on the data value of Q being 1

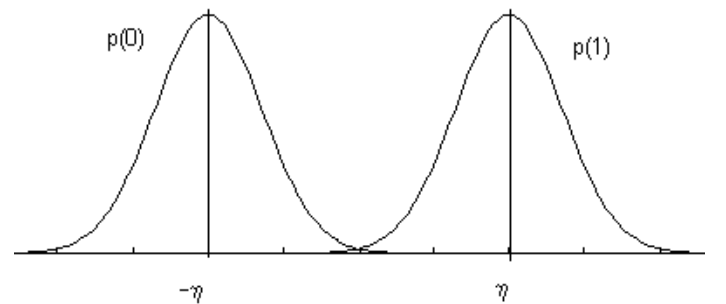
$$P_C = \cosh(2\eta(I_0 + Q_1))\cosh(2\eta(I_1 - Q_0)) + \cosh(2\eta(I_0 - I_1))\cosh(2\eta(Q_0 - Q_1)) \quad (11)$$

$P_D$  is the probability of vector pair in the IQ plane conditioned on the data value of Q being 0

$$P_D = \cosh(2\eta(I_0 - Q_1))\cosh(2\eta(I_1 + Q_0)) + \cosh(2\eta(I_0 + I_1))\cosh(2\eta(Q_0 + Q_1)) \quad (12)$$

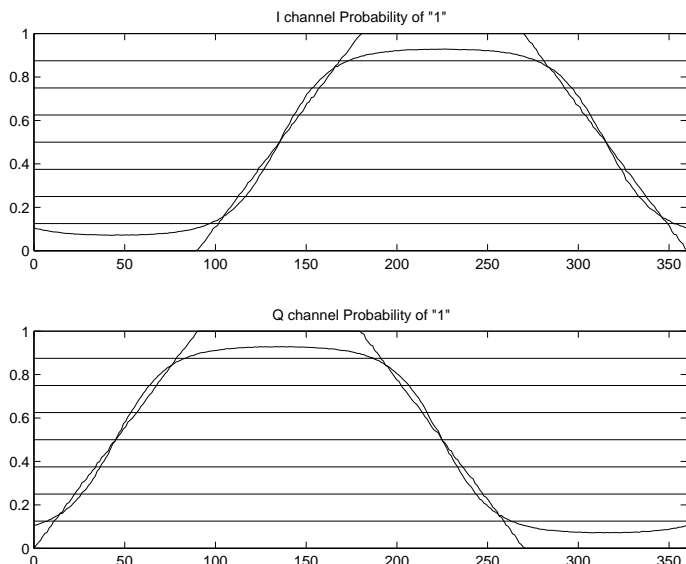
Once the floating point probability (8) is computed it could then be quantized to a 3-bit soft decision.

The expression above includes the term,  $\eta$ , which is the expected value of a one or a zero. The distribution of the data decisions and their relationship to  $\eta$  is shown in Figure 6.



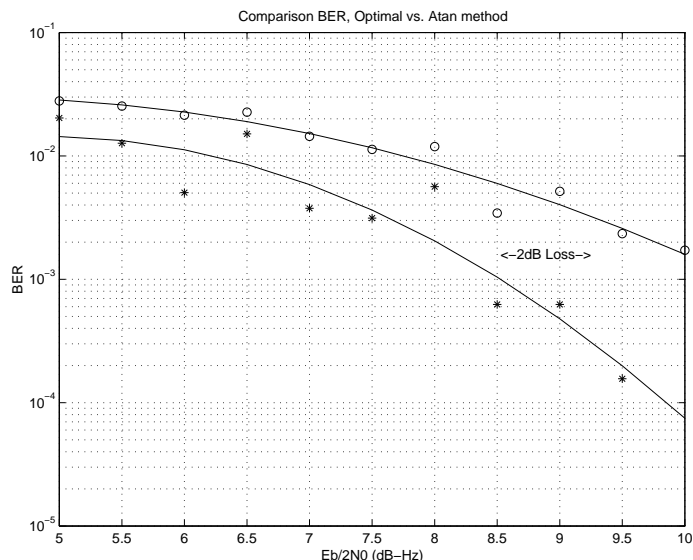
**Figure 6 Expected Value of "1" or "0"**

First it is beneficial to compare the results of the two methods as the differentially encoded data traverses the unit circle. The probability of “1” or a “0” for I and Q is show for each of the methods in Figure 7.



**Figure 7 Output as Data Traverses Unit Circle**

It’s important to note that during the transition regions the two methods are nearly identical in terms of mapping to soft-bits. The horizontal lines on the graphs show the quantization intervals for 3-bit soft decisions. In this benign case the soft-bit output for the given input would be identical. The next question to ask is the response of the algorithms in the presence of zero mean additive Gaussian noise. It comes back to the question what’s the implementation loss of one method versus the other. BER curves were generated via Monte Carlo simulation. The simulation band-limited the receive signal as would happen in any real-world transponder. Figure 8 shows the result of the simulation.



**Figure 8 BER Comparison**

The results show a disappointing 1.5 – 2.0 dB implementation loss for the arc-tangent method versus the optimal method. DSP clock cycles are precious but even more so modems respectable implementation losses.

**Conclusion**

DPSK modulation, by differencing phase change between consecutive signals can possibly reduce the effects of fading components however in the presence of zero-mean additive white noise one can expect between 1.5 to 2 dB of implementation loss in using the arc-tangent method of differentially decoding a DEQPSK signal. If ones design can afford the additional processor burden of computing 4 hyperbolic cosines per baud-time (versus a single atan2 per baud time) the optimal method shows offers distinct advantage.

**References**

- [1] ADSP 21000 Family Applications Handbook, Analog Devices, p 27
- [2] Soft-Decision Method for DEQPSK Tech Memo, Mike Durkin 4/7/99